

5.8 Radical Equations and Inequalities

radical equations--equation with a radical containing variables in the radicand

ex: $\sqrt{2x-1} = 3$

$$2x-1 = 9$$

$$2x = 10$$

$$x = 5 \checkmark$$

Steps

1. Isolate the radical
2. "Square" both sides
3. Solve for x
4. Check

ex: $2\sqrt[3]{x} - 1 = 3$

$$+1 \quad +1$$

$$2\sqrt[3]{x} = 4$$

$$\sqrt[3]{x} = 2$$

$$x = 8 \checkmark$$

4 ~~Extraneous~~

ex:

$$3x - 5\sqrt{x} = 2$$

$$\boxed{4}$$

$$(3x-2)^2 = (5\sqrt{x})^2$$

$$(3x-2)(3x-2)$$

$$9x^2 - 12x + 4 = 25x$$

$$9x^2 - 37x + 4 = 0$$

$$9x^2 - x - 36x + 4 = 0$$

$$x(9x-1) - 4(9x-1)$$

$$(x-4)(9x-1) = 0$$

$$x-4=0 \quad 9x-1=0$$

$$x=4$$

$$x = \frac{1}{9}$$

$$\begin{array}{r} 36 \\ -1 \times -36 \\ \hline -37 \end{array}$$

Do: $\{3, 4\}$

$$\sqrt{7x - 12} = x$$

$$7x - 12 = x^2$$

$$0 = x^2 - 7x + 12$$

$$(x-3)(x-4)$$

~~$\begin{array}{r} 12 \\ -3 \quad -4 \\ \hline -7 \end{array}$~~

Double $\sqrt{\frac{4}{9} + \frac{45}{9}} = \frac{4}{3} + 1 \checkmark$

$$\sqrt[ex]{2x+5}^2 = (2\sqrt{2x+1})^2$$

$$2x+5 = (2\sqrt{2x+1})(2\sqrt{2x+1})$$

$$4 \cdot 2x$$

$$2x+5 = 8x + 4\sqrt{2x} + 1$$

$$(-6x+4) = 4\sqrt{2x}$$

$$(-3x+2)^2 = (2\sqrt{2x})^2 (\div 2)$$

$$9x^2 - 12x + 4 = 8x$$

$$9x^2 - 20x + 4 = 0$$

$$(9x-2)(x-2) = 0 \quad \left(x = \frac{2}{9}\right) x \checkmark$$

Inequalities \mathbb{R}
ex: Radicand ≥ 0

$$\sqrt{3x+6} + 2 \leq 5$$

$$\sqrt{3x+6}^2 \leq 3^2$$

$$3x+6 \leq 9$$

$$3x \leq 3$$

$$x \leq 1$$

$$3x+6 \geq 0$$

$$3x \geq -6$$

$$x \geq -2$$

$$-2 \leq x \leq 1$$

Check 0 $\sqrt{6} + 2 \leq 5 \checkmark$
Check 2 $\sqrt{12} + 2 \leq 5 \checkmark$

$$x \geq 3$$

$$x \geq 2$$

$$x \geq 2$$